Cambright Solved Paper

Tags	2023, Additional Math, CIE IGCSE, May/June, P1, V1
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Status	Done

1 **(a)** Write $5x^2 - 14x + 8$ in the form $a(x+b)^2 + c$, where a, b and c are constants to be found. [3]

$$5(x^2 - \frac{14x}{5}) + 8$$
$$= 5[(x - \frac{7}{5})^2 - (\frac{7}{5})^2] + 8$$
$$= 5(x - \frac{7}{5})^2 - 5 \times \frac{49}{25} + 8$$
$$= 5(x - \frac{7}{5})^2 - \frac{49}{5} + 8$$
$$= 5(x - \frac{7}{5})^2 - \frac{9}{5}$$

(b) Hence write down the coordinates of the stationary point on the curve $y = 5x^2 - 14x + 8$. [2]

Stationary point occurs when the square is equal to 0

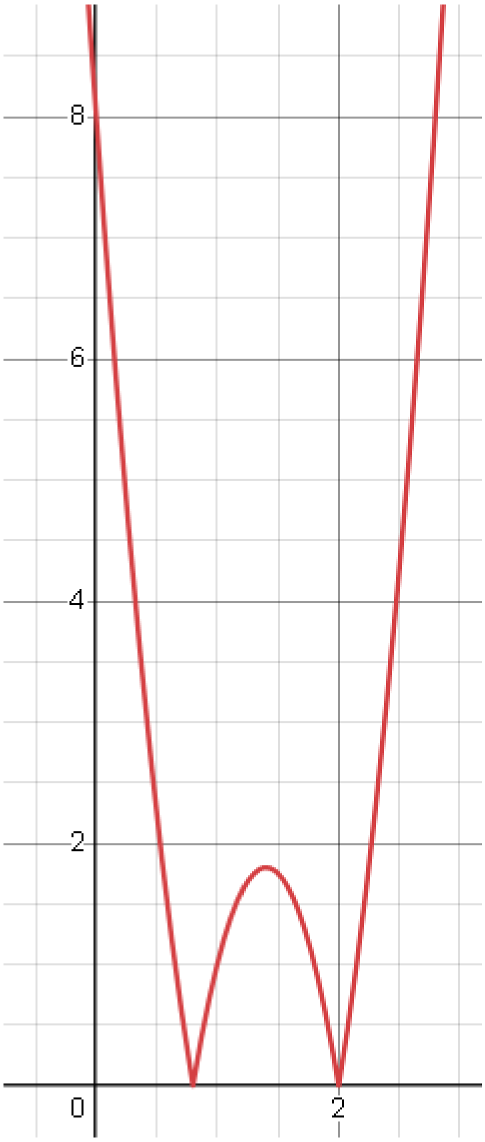
When $x = \frac{7}{5},\ 5(x - \frac{7}{5})^2 - \frac{9}{5} = 0$ and $y = -\frac{9}{5}$

Stationary point: $(\frac{7}{5},\ -\frac{9}{5})$

(c) On the axes below, sketch the graph of $y = |5x^2 - 14x + 8|$, stating the coordinates of the points where the graph meets the coordinate axes. [3]

When $x = 0,\ y = 8$ (y-intercept)

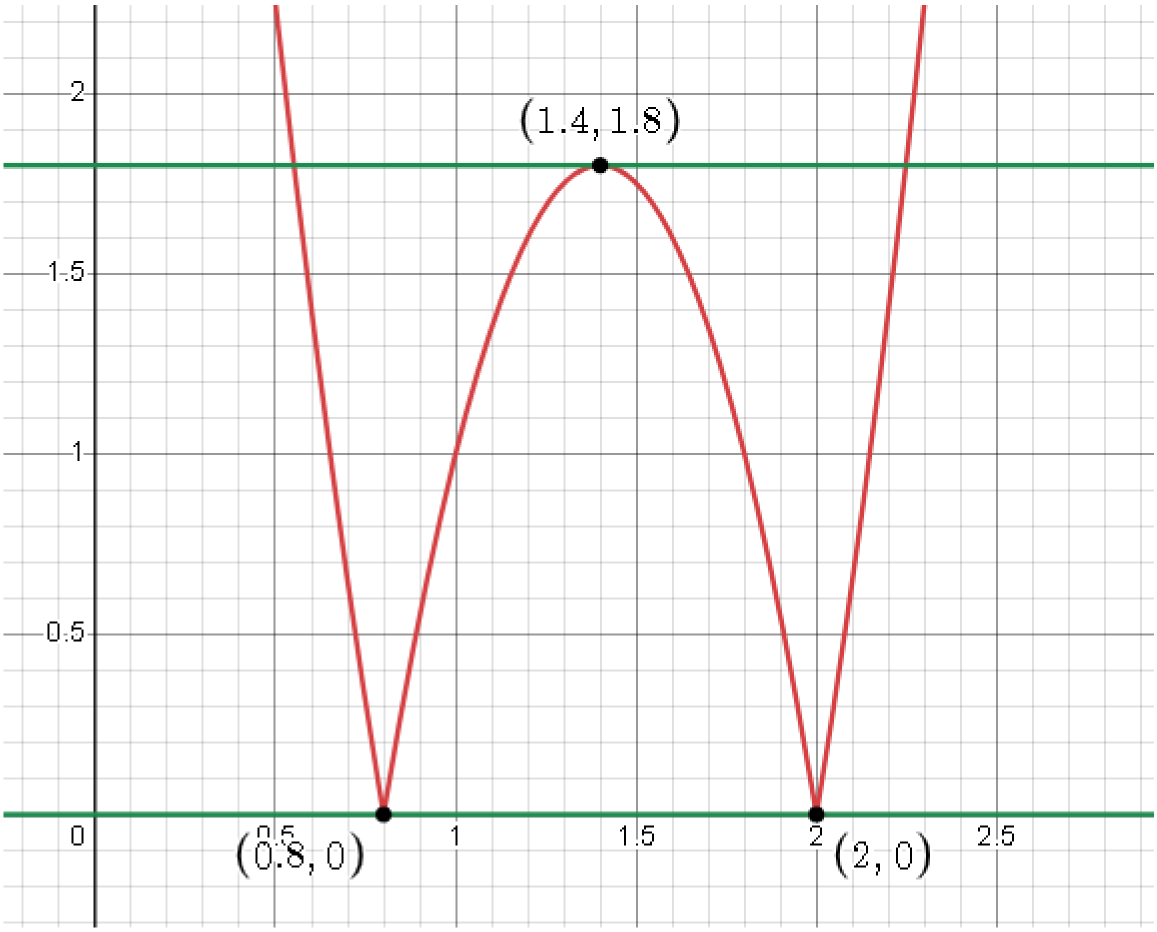
When $y = 0,\ x = 2\ or\ x = \frac{4}{5}(0.8)$



https://www.desmos.com/calculator/cdenwiy5gt

Make sure to label the intercepting points!!

(d) Write down the range of values of k for which the equation $\left|5x^2 - 14x + 8\right| = k$ has 4 distinct roots. [2]



https://www.desmos.com/calculator/cdenwiy5gt

Between the 2 green lines, a line k will have 4 roots.

$$0 < k < \frac{9}{5}$$

2 The polynomial p is such that $p(x) = ax^3 + 7x^2 + bx + c$, where a, b and c are integers.

(a) Given that $p''\left(\frac{1}{2}\right) = 32$, show that $a = 6$. [2]

$$p'(x) = 3ax^2 + 14x + b$$

$$p''(x) = 6ax + 14$$

$$Since\ p''(\frac{1}{2}) = 32,$$

$$6a \times \frac{1}{2} + 14 = 32$$

$$3a = 18$$

$$a = 6$$

(b) Given that $p(x)$ has a factor of $3x-4$ and a remainder of 7 when divided by $x+1$, find the values of b and c. [4]

Since 3x - 4 is a factor, $p(\frac{4}{3}) = 0$

$6(\frac{4}{3})^3 + 7(\frac{4}{3})^2 + b(\frac{4}{3}) + c = 0$

$\frac{128}{9} + \frac{112}{9} + b(\frac{4}{3}) + c = 0$

$\frac{80}{3} + b(\frac{4}{3}) + c = 0$

$80 + 4b + 3c = 0$

$p(-1) = 7$

$6(-1)^3 + 7(-1)^2 + b(-1) + c = 7$

$-6 + 7 - b + c = 7$

$-b + c = 6$

$c = 6 + b$

$Sub\, c = 6 + b\; into\; 80 + 4b + 3c = 0$

$80 + 4b + 3(6 + b) = 0$

$80 + 4b + 18 + 3b = 0$

$98 + 7b = 0$

$7b = -98$

$b = -14$

$Sub\, b = -14\; in\; c = 6 + b$

$c = 6 + (-14)$

$c = -8$

(c) Write $p(x)$ in the form $(3x-4)q(x)$, where $q(x)$ is a quadratic factor. [2]

You can use long division or the synthetic method

$$
\begin{array}{r|l}
 & 2x^2+5x+2 \\
\hline
3x-4 & 6x^3+7x^2-14x+8 \\
 & 6x^3-8x^2 \\
\hline
 & -15x^2-14x \\
 & -15x^2-20x \\
\hline
 & 6x+8 \\
 & -6x+8 \\
\hline
 & 0
\end{array}
$$

$p(x) = (3x-4)(2x^2+5x+2)$

(d) Hence write p(x) as a product of linear factors with integer coefficients. [1]

Using a calculator, we get $p(x) = (3x-4)(2x+1)(x+2)$

3 The points A and B have coordinates $(2, 5)$ and $(10, -15)$ respectively. The point P lies on the perpendicular bisector of the line AB. The y-coordinate of P is -9.

(a) Find the x-coordinate of P. [5]

$Midpoint\ AB = (\frac{2+10}{2}, \ \frac{5-15}{2}) = (6, \ -5)$

$gradient\ AB = \frac{rise}{run} = \frac{15-5}{10-2} = -\frac{5}{2}$

In perpendicular lines, $m_1 m_2 = -1$

$-\frac{5}{2}m_2 = -1$

$m_2 = \frac{2}{5}$

Now, we find the perpendicular bisector's equation, we can use the point slope form

$y-(-5) = \frac{2}{5}(x-6)$

$y+5 = \frac{2}{5}(x-6)$

The y-coordinate of P is given as $y = -9$

$-9+5 = \frac{2}{5}(x-6)$

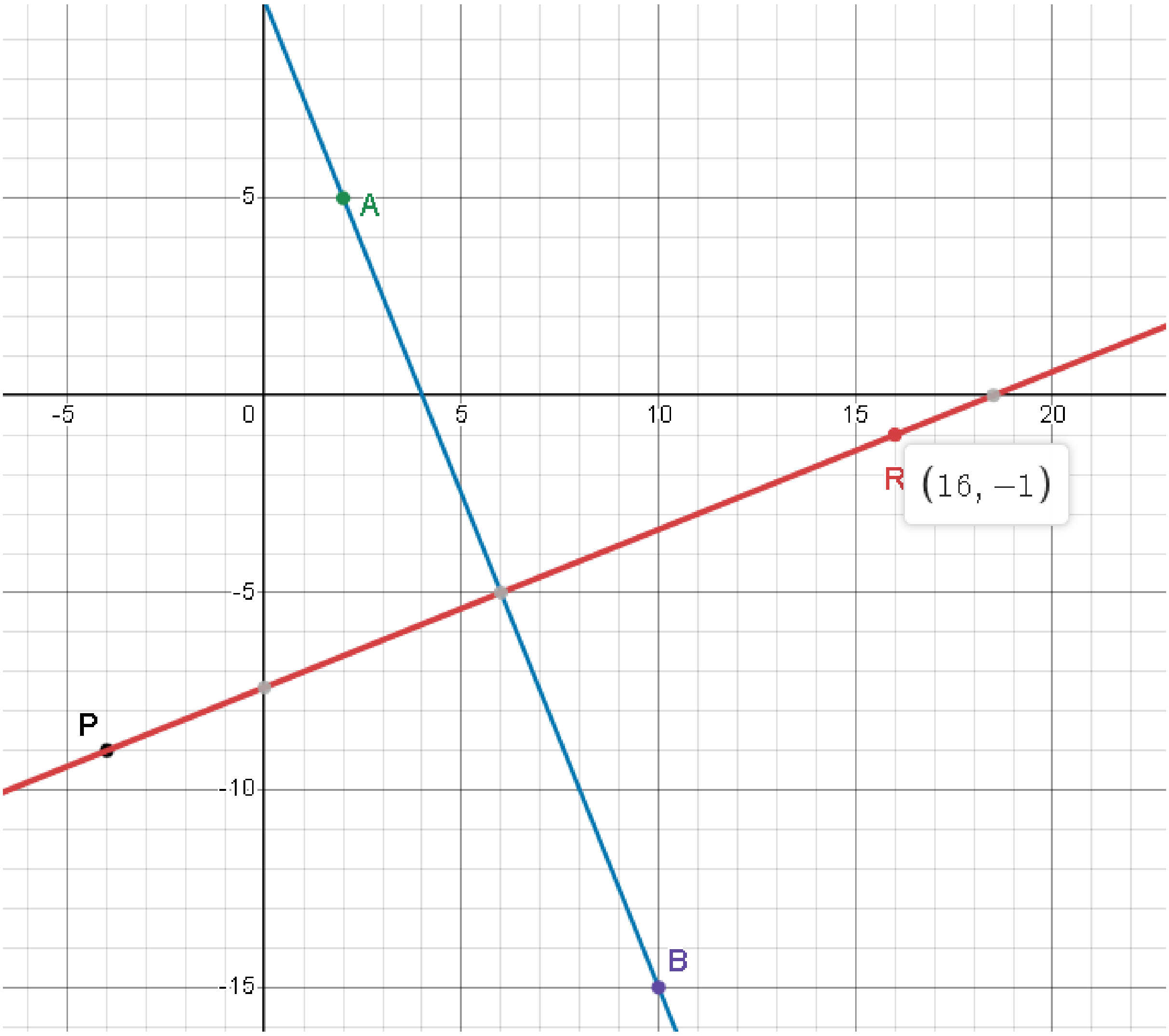
$$-4 = \frac{2}{5}(x-6)$$

$$-10 = x - 6$$

$$x = -4$$

(b) The point R is the reflection of P in the line AB. Find the coordinates of R. [2]

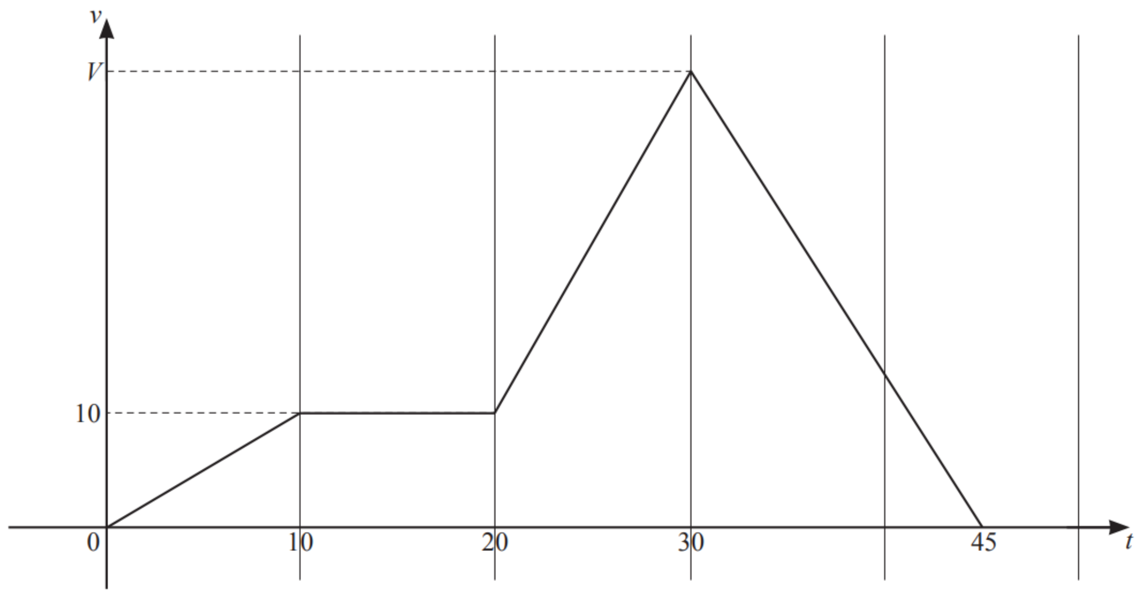
Using either a graph of symmetry, we get $R(16, -1)$



https://www.desmos.com/calculator/vuvzcgjah5 Graph method

Symmetry method: From P to midpoint, there is an x-translation of +10 and y-translation of +4, therefore $R(6+10,\ -5+4) = R(16,\ -1)$

4



The diagram shows the velocity–time graph for a particle travelling in a straight line with velocity, $v\,\mathrm{ms}^{-1}$, at time t seconds. When $t = 30$ the velocity of the particle is $V\,\mathrm{ms}^{-1}$. The particle travels 800 metres in 45 seconds.

(a) Find the value of V. [2]

The area under the graph of a velocity-time graph is the distance travelled. To find the area, break up the shape into a triangle, a rectangle, a trapezium, and a triangle (from left to right).

$$Area\ under\ graph = (\frac{1}{2} \times 10 \times 10) + (10 \times 10) + (\frac{10+V}{2} \times 10) + (\frac{1}{2} \times V \times 15)$$

$$800 = 50 + 100 + 50 + 5V + \frac{15V}{2}$$

$$\frac{25V}{2} = 600$$

$$V = 48$$

(b) Find the acceleration of the particle when $t = 35$. [2]

Acceleration is the gradient in a velocity-time graph

Since we don't know the velocity at t = 35, we will use the start-decelerating velocity at t = 30 and the final resting velocity at t = 45.

$$a = \frac{0-48}{45-30}$$

$$a = -\frac{16}{5}\ ms^{-2}$$

5 DO NOT USE A CALCULATOR IN THIS QUESTION.

In this question, all lengths are in centimetres.

(a) You are given that $\cos 120° = -\frac{1}{2}$, $\sin 120° = \frac{\sqrt{3}}{2}$ and $\tan 120° = -\sqrt{3}$.

In the triangle ABC, $AB = 5\sqrt{3} - 6$, $BC = 5\sqrt{3} + 6$ and angle $ABC = 120°$. Find AC, giving your answer in the form $a\sqrt{b}$ where a and b are integers greater than 1. [4]

Using the law of cosines:

$$AC^2 = AB^2 + BC^2 - 2(AB)(BC)\cos ABC$$

$$AC^2 = (5\sqrt{3} - 6)^2 + (5\sqrt{3} + 6)^2 - 2(5\sqrt{3} - 6)(5\sqrt{3} + 6)\cos 120^\circ$$

$$\cos 120^\circ = -\frac{1}{2}$$

$$AC^2 = (75 - 60\sqrt{3} + 36) + (75 + 60\sqrt{3} + 36) - \cancel{2}(75 - 36) \times -\frac{1}{\cancel{2}}$$

$$AC^2 = 261$$

$$AC = \sqrt{261}$$

$$AC = \sqrt{9 \times 29} = 3\sqrt{29}$$

(b) You are given that $\cos 30° = \frac{\sqrt{3}}{2}$, $\sin 30° = \frac{1}{2}$ and $\tan 30° = \frac{1}{\sqrt{3}}$.

In the triangle PQR, $PQ = 3 + 2\sqrt{5}$ and angle $PQR = 30°$. Given that the area of this triangle is $\frac{2+5\sqrt{5}}{4}$, find QR, giving your answer in the form $c + d\sqrt{5}$, where c and d are integers. [4]

$$Triangle\ area = \frac{1}{2}ab\ \sin C$$

Here, a is PQ, sin C is PQR and the area is given.

$$\frac{2+5\sqrt{5}}{4} = \frac{1}{2}(3 + 2\sqrt{5})(QR)\ \sin 30^\circ$$

$$\sin 30^\circ = \frac{1}{2}$$

$$\frac{2+5\sqrt{5}}{4} = \frac{1}{2}(3 + 2\sqrt{5})(QR) \times \frac{1}{2}$$

$$2 + \sqrt{5} = (3 + 2\sqrt{5})QR$$

$$QR = \frac{2+\sqrt{5}}{3+2\sqrt{5}}$$

Rationalize

$$QR = \frac{2+\sqrt{5}}{3+2\sqrt{5}} \times \frac{3-2\sqrt{5}}{3-2\sqrt{5}}$$

$$QR = \frac{6-4\sqrt{5}+15\sqrt{5}-50}{9-20}$$

$$QR = \frac{-44+11\sqrt{5}}{-11}$$

$$QR = 4 - \sqrt{5}$$

6 **(a)** Show that $\frac{\cot\theta + \tan\theta}{\sec\theta} = \operatorname{cosec}\theta$. [4]

$$\cot\theta = \frac{\cos\theta}{\sin\theta},\ \tan\theta = \frac{\sin\theta}{\cos\theta},\ \sec\theta = \frac{1}{\cos\theta}$$

$$\frac{\frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos\theta}} = \frac{\cos^2\theta \sin^2\theta}{\cancel{\cos\theta}\sin\theta} \times \cancel{\cos\theta}$$

$$\cos^2\theta + \sin^2\theta = 1$$

$$\frac{1}{\sin\theta} = \operatorname{cosec}\theta$$

(b) Hence solve the equation $\left(\frac{\cot\frac{\phi}{3} + \tan\frac{\phi}{3}}{\sec\frac{\phi}{3}}\right)^2 = 2$, for $-540° < \phi < 540°$. [6]

Substitute $\frac{1}{\sin\frac{\phi}{3}}$ into the bracket

$$(\frac{1}{\sin\frac{\phi}{3}})^2 = 2$$

$$\frac{1}{\sin\frac{\phi}{3}} = \pm\sqrt{2}$$

$$\sin\frac{\phi}{3} = \pm\frac{1}{\sqrt{2}} = \pm\frac{\sqrt{2}}{2}$$

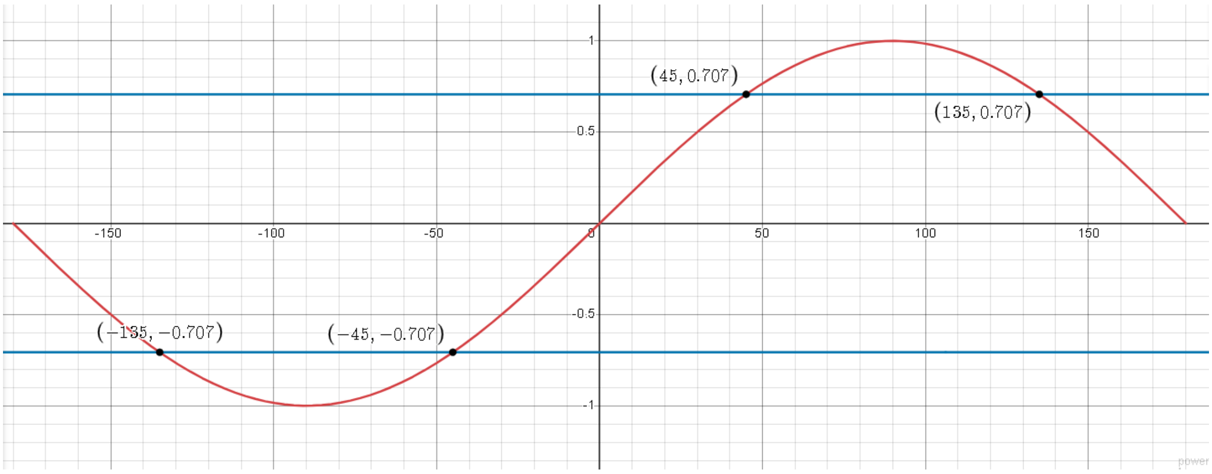
$$let\ x = \frac{\phi}{3},\ \ \phi = 3x$$

$$-540^\circ < \phi < 540^\circ$$

$-540^\circ < 3x < 540^\circ$

$-180^\circ < x < 180^\circ$

Now we can graph the function of $\sin x = \pm\frac{\sqrt{2}}{2}\ with\ -180^\circ < x < 180^\circ$



https://www.desmos.com/calculator/mtlbaqygxs

Here, we can see 4 points which solve our equation. $x = -135^\circ,\ -45^\circ,\ 45^\circ,\ and\ 135^\circ$

Substituting these values back into $\phi = 3x$, we get

$\phi = -405^\circ,\ -135^\circ,\ 135^\circ,\ and\ 405^\circ$

7 **(a)** A team of 8 people is to be chosen from a group of 15 people.

(i) Find the number of different teams that can be chosen. [1]

To choose 8 people from 15, we use the combination formula

$$15C8 = 6435$$

(ii) Find the number of different teams that can be chosen if the group of 15 people contains a family of 4 people who must be kept together. [3]

Case 1: include the family of 4, there will be 4 people left to choose from 11 people remaining.

$$11C4 = 330$$

Case 2: don't include the family of 4, there will 8 people to choose from 11 people remaining.

$$11C8 = 165$$

$$Total \ = 330 + 165 = 495$$

(b) Given that $(n+9) \times {}^{n}P_{10} = (n^2 + 243) \times {}^{n-1}P_9$, find the value of n. [3]

$$(n+9) \times \frac{n!}{(n-10)!} = (n^2 + 243) \times \frac{(n-1)!}{(n-1-9)!}$$

$$(n+9) \times \frac{n \times \cancel{(n-1)!}}{\cancel{(n-10)!}} = (n^2 + 243) \times \frac{\cancel{(n-1)!}}{\cancel{(n-10)!}}$$

$$(n+9) \times n = n^2 + 243$$

$$n^2 + 9n = n^2 + 243$$

$$9n = 243$$

$$n = 27$$

8 A curve has the equation $y = \frac{(3x-4)^{\frac{1}{3}}}{2x+1}$.

(a) Show that $\frac{dy}{dx} = \frac{Ax+B}{(2x+1)^2(3x-4)^{\frac{2}{3}}}$, where A and B are integers to be found. [5]

Quotient rule: if $y = \frac{f}{g}$, $\frac{dy}{dx} = \frac{gf' - fg'}{g^2}$

$f = (3x-4)^{\frac{1}{3}},\ f' = \frac{1}{3} \times (3x-4)^{\frac{-2}{3}} \times 3 = (3x-4)^{\frac{-2}{3}}$

$g = 2x+1,\ g' = 2$

$$\frac{dy}{dx} = \frac{(2x+1)(3x-4)^{\frac{-2}{3}} - 2(3x-4)^{\frac{1}{3}}}{(2x+1)^2}$$

Factor out $\frac{(3x-4)^{\frac{-2}{3}}}{(2x+1)^2}$

$$\frac{dy}{dx} = \frac{(3x-4)^{\frac{-2}{3}}}{(2x+1)^2} \times [(2x+1) - 2(3x-4)]$$

$$\frac{dy}{dx} = \frac{(3x-4)^{\frac{-2}{3}}}{(2x+1)^2} \times (2x+1-6x+8)$$

$$\frac{dy}{dx} = \frac{(3x-4)^{\frac{-2}{3}}}{(2x+1)^2} \times (9-4x)$$

Rearrange

$$\frac{dy}{dx} = \frac{(9-4x)}{(2x+1)^2(3x-4)^{\frac{2}{3}}}$$

(b) Find the coordinates of the stationary point on the curve. [2]

We can find the stationary point when $\frac{dy}{dx} = 0$

$$\frac{(9-4x)}{(2x+1)^2(3x-4)^{\frac{2}{3}}} = 0$$

$9 - 4x = 0$

$4x = 9$

$x = \frac{9}{4}\ or\ 2.25$

Substitute $x = \frac{9}{4}\ or\ 2.25$ into $y = \frac{(3x-4)^{\frac{1}{3}}}{2x+1}$

$y = \frac{(3(2.25)-4)^{\frac{1}{3}}}{2(2.25)+1} = 0.25473...$

$y = 0.255$

Stationary point (2.25, 0.255)

9 (a) The first three terms of an arithmetic progression are $\ln q$, $\ln q^4$ and $\ln q^7$, where q is a positive constant. The sum to n terms of this progression is $4845 \ln q$. Find the value of n. [3]

Common difference $d = u_2 - u_1 = \ln q^4 - \ln q = 4 \ln q - \ln q = 3 \ln q$

$S_n = \frac{n}{2}\{2a + (n-1)d\}$

$4845 \ln q = \frac{n}{2}\{2 \ln q + (n-1)(3 \ln q)\}$

$9690 \ln q = n\{2 \ln q + 3n \ln q - 3 \ln q\}$

$9690 \ln q = n\{3n \ln q - \ln q\}$

$9690 \ln q = 3n^2 \ln q - n \ln q$

$3n^2 \ln q - n \ln q - 9690 \ln q = 0$

Remove ln q from both sides

$3n^2 - n - 9690 = 0$

$(n-57)(3n+170) = 0$

$n = 57\ or\ n = -\frac{170}{3}$

n will always be positive so our final answer is $n = 57$

(b) The first three terms of a geometric progression are p^{3x}, p^{x} and p^{-x}, where p is a positive integer. Find the nth term of this progression giving your answer in the form $p^{(a+bn)x}$. [3]

Common ratio $r = \frac{u_2}{u_1} = \frac{p^x}{p^{3x}} = p^{-2x}$

$n^{th}\ term = ar^{n-1}$

$n^{th}\ term = p^{3x}(p^{-2x})^{n-1}$

$n^{th}\ term = p^{3x}(p^{-2nx+2x})$

$n^{th}\ term = p^{3x-2nx+2x} = p^{5x-2nx}$

$n^{th}\ term = p^{(5-2n)x}$

(c) The first three terms of a different geometric progression are $\frac{4}{3}\cos^2 3\theta$, $\frac{16}{9}\cos^4 3\theta$ and $\frac{64}{27}\cos^6 3\theta$, for $0 < \theta < \frac{\pi}{3}$. Find the set of values of θ for which this progression has a sum to infinity. [5]

Common ratio $r = \frac{u_2}{u_1} = \frac{\frac{16}{9}\cos^4 3\theta}{\frac{4}{3}\cos^2 3\theta} = \frac{4}{3}\cos^2 3\theta$

For a sum to infinity, $-1 < r < 1$, here r is $\frac{4}{3}\cos^2 3\theta$ and has a square in it, so any value will be positive, between 0 and 1. $0 < r < 1$

For this, we only need to consider $\frac{4}{3}\cos^2 3\theta < 1$

$\cos^2 3\theta < \frac{3}{4}$

$\cos 3\theta < \pm\sqrt{\frac{3}{4}}$

$\cos 3\theta < \pm\frac{\sqrt{3}}{4}$

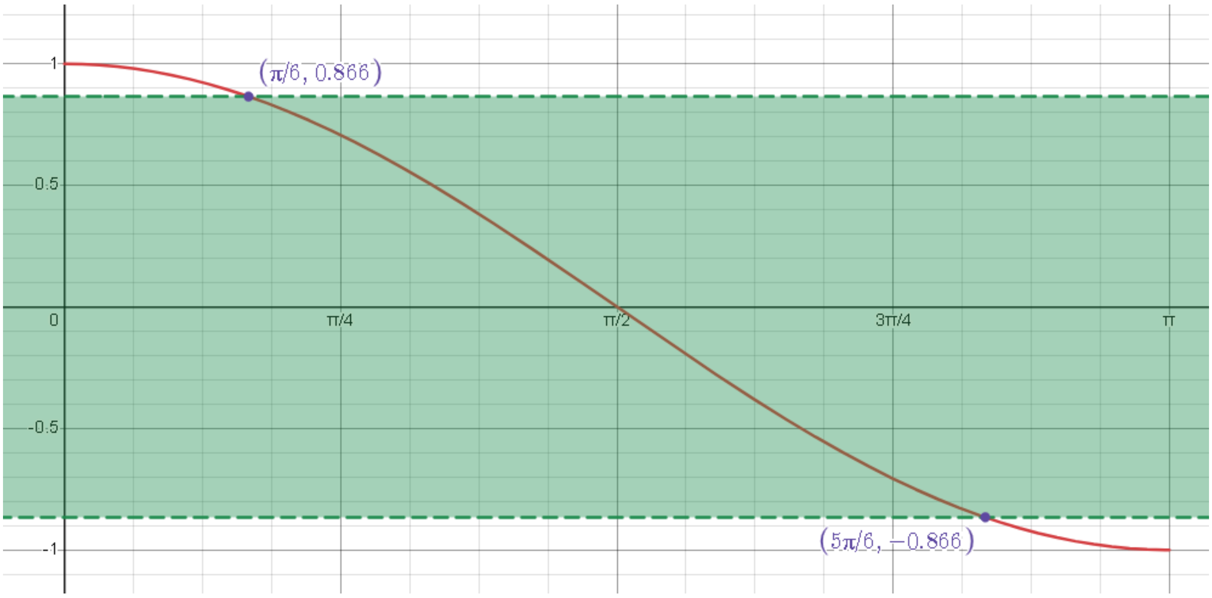
Let $x = 3\theta,\ \theta = \frac{x}{3}$

$0 < \theta < \frac{\pi}{3}$

$0 < \frac{x}{3} < \frac{\pi}{3}$

$0 < x < \pi$

Now, we can graph the function of $\cos x < \pm\frac{\sqrt{3}}{2}$ along with our calculator to find each values



https://www.desmos.com/calculator/2dwvpkiwk5

Here, we can see that between $x = \frac{\pi}{6}$ and $x = \frac{5\pi}{6}$, we get our range of solutions.

So $\frac{\pi}{6} < x < \frac{5\pi}{6}$

Substitute $x = 3\theta$

$\frac{\pi}{6} < 3\theta < \frac{5\pi}{6}$

$\frac{\pi}{18} < \theta < \frac{5\pi}{18}$

10 It is given that $y = (3x+1)^2 \ln(3x+1)$.

(a) Find $\frac{dy}{dx}$. [3]

$\frac{d(3x+1)^2}{dx} = 2(3x+1) \times 3 = 6(3x+1)$

$\frac{d(\ln(3x+1))}{dx} = \frac{3}{3x+1}$

-

$$\frac{dy}{dx} = 6(3x+1) \times \ln(3x+1) + (3x+1)^2 \times \frac{3}{3x+1}$$
$$\frac{dy}{dx} = 6(3x+1) \times \ln(3x+1) + 3(3x+1)$$

(b) Hence find $\int (3x+1)\ln(3x+1)\,dx$. [4]

Since $\frac{dy}{dx} = 6(3x+1) \times \ln(3x+1) + 3(3x+1)$,
$\int 6(3x+1) \times \ln(3x+1) + 3(3x+1)dx = (3x+1)^2 \ln(3x+1) + c$

Here, we want to remove +3(3x + 1) and the coefficient 6
$\int 6(3x+1)\ln(3x+1)dx + \int 3(3x+1)dx = (3x+1)^2 \ln(3x+1) + c$

Subtract $\int 3(3x+1)dx$ from both sides
$\int 6(3x+1)\ln(3x+1)dx = (3x+1)^2 \ln(3x+1) - \int 3(3x+1)dx + c$
$\int 6(3x+1)\ln(3x+1)dx = (3x+1)^2 \ln(3x+1) - 3(\frac{3x^2}{2} + x) + c$
$\int 6(3x+1)\ln(3x+1)dx = (3x+1)^2 \ln(3x+1) - \frac{9x^2}{2} + 3x + c$

Using integral rules, we can move the coefficient 6 to the front of the integral
$6\int (3x+1)\ln(3x+1)dx = (3x+1)^2 \ln(3x+1) - \frac{9x^2}{2} + 3x + c$

Divide both sides by 6
$\int (3x+1)\ln(3x+1)dx = \frac{1}{6}(3x+1)^2 \ln(3x+1) - \frac{1}{6}(\frac{9x^2}{2} + 3x) + c$
$\int (3x+1)\ln(3x+1)dx = \frac{1}{6}(3x+1)^2 \ln(3x+1) - \frac{3x^2}{4} + \frac{x}{2} + c$

Additional notes

Websites and resources used:

- Desmos graphing calculator

If you find any errors or mistakes within this paper, please contact us and we will fix them as soon as possible.